Space-Time Compactification Induced By Lightlike Branes

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Abstract

The aim of the present paper is two-fold. First we describe the Lagrangian dynamics of a recently proposed new class of *lightlike* p-branes and their interactions with bulk space-time gravity and electromagnetism in a self-consistent manner. Next, we discuss the role of *lightlike* branes as natural candidates for *wormhole* "throats" and exemplify the latter by presenting an explicit construction of a new type of asymmetric wormhole solution where the *lightlike* brane connects a "right" universe with Reissner-Nordström geometry to a "left" Bertotti-Robinson universe with two compactified space dimensions.

Keywords: traversable wormholes; non-Nambu-Goto lightlike branes; dynamical brane tension; black hole's horizon "straddling"

1 Introduction

Lightlike branes (*LL-branes* for short) play an increasingly significant role in general relativity and modern non-perturbative string theory. Mathematically they represent singular null hypersurfaces in Riemannian space-time which provide dynamical description of various physically important cosmological and astrophysical phenomena such as:

(i) Impulsive lightlike signals arising in cataclysmic astrophysical events (supernovae, neutron star collisions) [1];

(ii) Dynamics of horizons in black hole physics – the so called "membrane paradigm" [2];

(iii) The thin-wall approach to domain walls coupled to gravity [3, 4, 5].

More recently, the relevance of *LL-branes* in the context of non-perturbative string theory has also been recognized, specifically, as the so called H-branes describing quantum horizons (black hole and cosmological) [6], as Penrose limits of baryonic D-branes [7], *etc* (see also Refs.[8]).

A characteristic feature of the formalism for *LL-branes* in the pioneering papers [3, 4, 5] in the context of gravity and cosmology is that they have been exclusively treated in a "phenomenological" manner, *i.e.*, without specifying an underlying Lagrangian dynamics from which they may originate. As a partial exception, in a more recent paper [9] brane actions in terms of their pertinent extrinsic geometry have been proposed which generically describe non-lightlike branes, whereas the lightlike branes are treated as a limiting case.

On the other hand, in the last few years we have proposed in a series of papers [10, 11, 12, 13] a new class of concise manifestly reparametrization invariant world-volume Lagrangian actions, providing a derivation from first principles of the *LL-brane* dynamics. The following characteristic features of the new *LL-branes* drastically distinguish them from ordinary Nambu-Goto branes:

(a) They describe intrinsically lightlike modes, whereas Nambu-Goto branes describe massive ones.

(b) The tension of the *LL-brane* arises as an *additional dynamical degree of freedom*, whereas Nambu-Goto brane tension is a given *ad hoc* constant. The latter characteristic feature significantly distinguishes our *LL-brane* models from the previously proposed *tensionless* p-branes (for a review, see Ref.[14]). The latter rather resemble p-dimensional continuous distributions of independent massless point-particles without cohesion among the latter.

(c) Consistency of *LL-brane* dynamics in a spherically or axially symmetric gravitational background of codimension one requires the presence of an event horizon which is automatically occupied by the *LL-brane* ("horizon straddling" according to the terminology of Ref.[4]).

(d) When the *LL-brane* moves as a *test* brane in spherically or axially symmetric gravitational backgrounds its dynamical tension exhibits exponential "inflation/deflation" time behavior [11] – an effect

similar to the "mass inflation" effect around black hole horizons [15].

An intriguing novel application of *LL-branes* as natural self-consistent gravitational sources for *wormhole* space-times has been developed in a series of recent papers [12, 13, 16, 17].

Before proceeding let us recall that the concept of "wormhole space-time" was born in the classic work of Einstein and Rosen [18], where they considered matching along the horizon of two identical copies of the exterior Schwarzschild space-time region (subsequently called *Einstein-Rosen "bridge"*). Another corner stone in wormhole physics is the seminal work of Morris and Thorne [19], who studied for the first time *traversable Lorentzian wormholes*.

In what follows, when discussing wormholes we will have in mind the physically important class of "thin-shell" traversable Lorentzian wormholes first introduced by Visser [20, 21]. For a comprehensive review of wormhole space-times, see Refs.[21, 22].

In our earlier work [12, 13, 16, 17] we have constructed various types of wormhole solutions in self-consistent systems of bulk gravity and bulk gauge fields (Maxwell and Kalb-Ramond) coupled to *LL-branes* where the latter provide the appropriate stress energy tensors, electric currents and dynamically generated space-varying cosmological constant terms consistently derived from well-defined world-volume *LL-brane* Lagrangian actions.

The original Einstein-Rosen "bridge" manifold [18] appears as a particular case of the construction of spherically symmetric wormholes produced by *LL-branes* as gravitational sources occupying the wormhole throats (Refs.[16, 13]). Thus, we are lead to the important conclusion that consistency of Einstein equations of motion yielding the original Einstein-Rosen "bridge" as well-defined solution necessarily requires the presence of *LL-brane* energy-momentum tensor as a source on the right hand side.

More complicated examples of spherically and axially symmetric wormholes with Reissner-Nordström and rotating cylindrical geometry, respectively, have been explicitly constructed via *LL-branes* in Refs.[12, 13]. Namely, two copies of the exterior space-time region of a Reissner-Nordström or rotating cylindrical black hole, respectively, are matched via *LL-brane* along what used to be the outer horizon of the respective full black hole space-time manifold. In this way one obtains a wormhole solution which combines the features of the Einstein-Rosen "bridge" on the one hand (with wormhole throat at horizon), and the features of Misner-Wheeler wormholes [23], *i.e.*, exhibiting the so called "charge without charge" phenomenon.

Recently the results of Refs.[12, 13] have been extended to the case of *asymmetric* wormholes, describing two "universes" with different spherically symmetric geometries of black hole type connected via a "throat" materialized by the pertinent gravitational source – an electrically charged *LL-brane*, sitting on their common horizon. As a result of the well-defined world-volume *LL-brane* dynamics coupled self-consistently to gravity and bulk space-time gauge fields, it creates a "left universe" comprising the exterior Schwarzschild-de-Sitter space-time region beyond the Schwarzschild horizon and where the cosmological constant is dynamically generated, and a "right universe" comprising the exterior Reissner-Nordström region beyond the outer Reissner-Nordström horizon with dynamically generated Coulomb field-strength. Both "universes" are glued together by the *LL-brane* occupying their common horizon. Similarly, the *LL-brane* can dynamically generate a non-zero cosmological constant in the "right universe", in which case it connects a purely Schwarzschild "left universe" with a Reissner-Nordström-de-Sitter "right universe".

In the present paper we will further broaden the application of *LL-branes* in the context of wormhole physics by constructing a new type of wormhole solution to Einstein-Maxwell equations describing a "right universe", which comprises the exterior Reissner-Nordström space-time region beyond the outer Reissner-Nordström horizon, connected through a "throat" materialized by a *LL-brane* with a "left universe" being a Bertotti-Robinson space-time with two compactified spatial dimensions [24] (see also [25]).

Let us note that previously the junction of a compactified space-time (of Bertotti-Robinson type) to an uncompactified space-time through a wormhole has been studied in a different setting using *time-like* matter on the junction hypersurface [26]. Also, in a different context a string-like (flux tube) object with similar features to Bertotti-Robinson solution has been constructed [27] which interpolates between uncompactified space-time regions.

2 World-Volume Formulation of Lightlike Brane Dynamics

There exist two equivalent *dual to each other* manifestly reparametrization invariant world-volume Lagrangian formulations of *LL-branes* [10, 11, 12, 13, 16, 28]. First, let us consider the Polyakov-type for-

mulation where the *LL-brane* world-volume action is given as:

$$S_{\rm LL} = \int d^{p+1}\sigma\,\Phi\left[-\frac{1}{2}\gamma^{ab}g_{ab} + L\big(F^2\big)\right] \; . \qquad (1)$$

Here the following notions and notations are used:

(a) Φ is alternative non-Riemannian integration measure density (volume form) on the p-brane world-volume manifold:

$$\Phi \equiv \frac{1}{(p+1)!}\varepsilon^{a_1\ldots a_{p+1}} H_{a_1\ldots a_{p+1}}(B) \; , \qquad (2)$$

$$H_{a_1\ldots a_{p+1}}(B) = (p+1)\partial_{[a_1}B_{a_2\ldots a_{p+1}]} \; , \qquad (3)$$

instead of the usual $\sqrt{-\gamma}$. Here $\varepsilon^{a_1\ldots a_{p+1}}$ is the alternating symbol ($\varepsilon^{01\ldots p}=1$), γ_{ab} $(a,b=0,1,\ldots,p)$ indicates the intrinsic Riemannian metric on the world-volume, and $\gamma = \det\|\gamma_{ab}\|$. $H_{a_1\ldots a_{p+1}}(B)$ denotes the field-strength of an auxiliary world-volume antisymmetric tensor gauge field $B_{a_1\ldots a_p}$ of rank p. As a special case one can build $H_{a_1\ldots a_{p+1}}$ in terms of $p+1$ auxiliary world-volume scalar fields $\left\{\varphi^I\right\}_{I=1}^{p+1}$:

$$H_{a_1\ldots a_{p+1}} = \varepsilon_{I_1\ldots I_{p+1}}\partial_{a_1}\varphi^{I_1}\ldots\partial_{a_{p+1}}\varphi^{I_{p+1}} \; . \qquad (4)$$

Note that γ_{ab} is *independent* of the auxiliary world-volume fields $B_{a_1\ldots a_p}$ or φ^I. The alternative non-Riemannian volume form (2) has been first introduced in the context of modified standard (non-lightlike) string and p-brane models in Refs.[29].

(b) $X^\mu(\sigma)$ are the p-brane embedding coordinates in the bulk D-dimensional space time with bulk Riemannian metric $G_{\mu\nu}(X)$ with $\mu,\nu = 0,1,\ldots,D-1$; $(\sigma) \equiv \left(\sigma^0 \equiv \tau, \sigma^i\right)$ with $i=1,\ldots,p$; $\partial_a \equiv \frac{\partial}{\partial\sigma^a}$.

(c) g_{ab} is the induced metric on world-volume:

$$g_{ab} \equiv \partial_a X^\mu \partial_b X^\nu G_{\mu\nu}(X) \; , \qquad (5)$$

which becomes *singular* on-shell (manifestation of the lightlike nature, cf. second Eq.(10) below).

(d) $L\big(F^2\big)$ is the Lagrangian density of another auxiliary $(p-1)$-rank antisymmetric tensor gauge field $A_{a_1\ldots a_{p-1}}$ on the world-volume with p-rank field-strength and its dual:

$$F_{a_1\ldots a_p} = p\partial_{[a_1}A_{a_2\ldots a_p]} \quad , \quad F^{*a} = \frac{1}{p!}\frac{\varepsilon^{aa_1\ldots a_p}}{\sqrt{-\gamma}}F_{a_1\ldots a_p} \; . \qquad (6)$$

$L\big(F^2\big)$ is *arbitrary* function of F^2 with the short-hand notation: $F^2 \equiv F_{a_1\ldots a_p}F_{b_1\ldots b_p}\gamma^{a_1b_1}\ldots\gamma^{a_pb_p}$.

Rewriting the action (1) in the following equivalent form:

$$S = -\int d^{p+1}\sigma\,\chi\sqrt{-\gamma}\Big[\frac{1}{2}\gamma^{ab}g_{ab} - L\big(F^2\big)\Big] \; ,$$

$$\chi \equiv \frac{\Phi}{\sqrt{-\gamma}} \qquad (7)$$

with Φ the same as in (2), we find that the composite field χ plays the role of a *dynamical (variable) brane tension*[1].

Let us now consider the equations of motion corresponding to (1) w.r.t. $B_{a_1\ldots a_p}$:

$$\partial_a\Big[\frac{1}{2}\gamma^{cd}g_{cd} - L(F^2)\Big] = 0 \;\; \to \;\; \frac{1}{2}\gamma^{cd}g_{cd} - L(F^2) = M \; , \qquad (8)$$

where M is an arbitrary integration constant. The equations of motion w.r.t. γ^{ab} read:

$$\frac{1}{2}g_{ab} - F^2L^\prime(F^2)\left[\gamma_{ab} - \frac{F^*_aF^*_b}{F^{*\,2}}\right] = 0 \; , \qquad (9)$$

where $F^{*\,a}$ is the dual field strength (6). Eqs.(9) can be viewed as p-brane analogues of the string Virasoro constraints.

Taking the trace in (9) and comparing with (8) implies the following crucial relation for the Lagrangian function $L\left(F^2\right)$: $L\big(F^2\big) - pF^2L^\prime\big(F^2\big) + M = 0$, which determines F^2 on-shell as certain function of the integration constant M (8), *i.e.* $F^2 = F^2(M) = \mathrm{const}$. Here and below $L^\prime(F^2)$ denotes derivative of $L(F^2)$ w.r.t. the argument F^2.

The next and most profound consequence of Eqs.(9) is that the induced metric (5) on the world-volume of the p-brane model (1) is *singular* on-shell (as opposed to the induced metric in the case of ordinary Nambu-Goto branes):

$$g_{ab}F^{*\,b} \equiv \partial_a X^\mu G_{\mu\nu}\left(\partial_b X^\nu F^{*\,b}\right) = 0 \; . \qquad (10)$$

Eq.(10) is the manifestation of the *lightlike* nature of the p-brane model (1) (or (7)), namely, the tangent vector to the world-volume $F^{*\,a}\partial_a X^\mu$ is *lightlike* w.r.t. metric of the embedding space-time.

Further, the equations of motion w.r.t. world-volume gauge field $A_{a_1\ldots a_{p-1}}$ (with χ as defined in (7) read:

$$\partial_{[a}\left(F^*_{b]}\,\chi\right) = 0 \; . \qquad (11)$$

Finally, the X^μ equations of motion produced by the (1) read:

$$\partial_a\left(\chi\sqrt{-\gamma}\gamma^{ab}\partial_b X^\mu\right) + \chi\sqrt{-\gamma}\gamma^{ab}\partial_a X^\nu\partial_b X^\lambda\Gamma^\mu_{\nu\lambda} = 0 \qquad (12)$$

where $\Gamma^\mu_{\nu\lambda} = \frac{1}{2}G^{\mu\kappa}\left(\partial_\nu G_{\kappa\lambda} + \partial_\lambda G_{\kappa\nu} - \partial_\kappa G_{\nu\lambda}\right)$ is the Christoffel connection for the external metric.

[1] The notion of dynamical brane tension has previously appeared in different contexts in Refs.[30].

Eq.(11) allows us to introduce the dual "gauge" potential u (dual w.r.t. world-volume gauge field $A_{a_1\ldots a_{p-1}}$ (6)) :

$$F^*_a = c_p\,\frac{1}{\chi}\partial_a u \quad , \quad c_p = \text{const} \; . \tag{13}$$

Relation (13) enables us to rewrite Eq.(9) (the lightlike constraint) in terms of the dual potential u in the form:

$$\gamma_{ab} = \frac{1}{2a_0}\,g_{ab} - \frac{(2a_0)^{p-2}}{\chi^2}\,\partial_a u \partial_b u$$
$$a_0 \equiv F^2 L'\left(F^2\right)\big|_{F^2=F^2(M)} = \text{const} \; . \tag{14}$$

($L'(F^2)$ denotes derivative of $L(F^2)$ w.r.t. the argument F^2). From (13) we obtain the relation:

$$\chi^2 = -(2a_0)^{p-2}\gamma^{ab}\partial_a u \partial_b u \; , \tag{15}$$

and the Bianchi identity $\nabla_a F^{*\,a} = 0$ becomes:

$$\partial_a\Big(\frac{1}{\chi}\sqrt{-\gamma}\gamma^{ab}\partial_b u\Big) = 0 \; . \tag{16}$$

It is straightforward to prove that the system of equations (12), (16) and (15) for (X^μ, u, χ), which are equivalent to the equations of motion (8)–(11),(12) resulting from the original Polyakov-type *LL-brane* action (1), can be equivalently derived from the following *dual* Nambu-Goto-type world-volume action:

$$S_{\rm NG} = -\int d^{p+1}\sigma\, T\sqrt{\Big|\,\det\|g_{ab} - \epsilon\frac{1}{T^2}\partial_a u \partial_b u\|\,\Big|} \; , \tag{17}$$

with $\epsilon = \pm 1$. Here again g_{ab} indicates the induced metric on the world-volume (5) and T is dynamical variable tension simply proportional to χ ($\chi^2 = (2a_0)^{p-1}T^2$with a_0 as in (14)). The choice of the sign in (17) does not have physical effect because of the non-dynamical nature of the u-field.

Henceforth we will stick to the Polyakov-type formulation of world-volume *LL-brane* dynamics since within this framework one can add in a natural way [10, 11, 12] couplings of the *LL-brane* to bulk space-time Maxwell $\mathcal{A}_\mu$ and Kalb-Ramond $\mathcal{A}_{\mu_1\ldots\mu_{D-1}}$ gauge fields (in the case of codimension one *LL-branes*, *i.e.*, for $D = (p+1)+1$):

$$\widetilde{S}_{\rm LL} = S_{\rm LL} - q\int d^{p+1}\sigma\,\varepsilon^{ab_1\ldots b_p}F_{b_1\ldots b_p}\partial_a X^\mu \mathcal{A}_\mu$$
$$-\frac{\beta}{(p+1)!}\int d^{p+1}\sigma\,\varepsilon^{a_1\ldots a_{p+1}}\partial_{a_1}X^{\mu_1}\ldots\partial_{a_{p+1}}X^{\mu_{p+1}}$$
$$\times\mathcal{A}_{\mu_1\ldots\mu_{p+1}} \tag{18}$$

with $S_{\rm LL}$ as in (1). The *LL-brane* constraint equations (8)–(9) are not affected by the bulk space-time gauge field couplings whereas Eqs.(11)–(12) acquire the form:

$$\partial_{[a}\left(F^*_{b]}\,\chi L'(F^2)\right) + \frac{q}{4}\partial_a X^\mu \partial_b X^\nu \mathcal{F}_{\mu\nu} = 0 \; ; \tag{19}$$

$$\partial_a\left(\chi\sqrt{-\gamma}\gamma^{ab}\partial_b X^\mu\right) + \chi\sqrt{-\gamma}\gamma^{ab}\partial_a X^\nu \partial_b X^\lambda \Gamma^\mu_{\nu\lambda}$$
$$-q\varepsilon^{ab_1\ldots b_p}F_{b_1\ldots b_p}\partial_a X^\nu \mathcal{F}_{\lambda\nu}G^{\lambda\mu}$$
$$-\frac{\beta}{(p+1)!}\varepsilon^{a_1\ldots a_{p+1}}\partial_{a_1}X^{\mu_1}\ldots\partial_{a_{p+1}}X^{\mu_{p+1}}$$
$$\times\mathcal{F}_{\lambda\mu_1\ldots\mu_{p+1}}G^{\lambda\mu} = 0 \; . \tag{20}$$

Here χ is the dynamical brane tension as in (7), $\mathcal{F}_{\mu\nu} = \partial_\mu \mathcal{A}_\nu - \partial_\nu \mathcal{A}_\mu$ and

$$\mathcal{F}_{\mu_1\ldots\mu_D} = D\partial_{[\mu_1}\mathcal{A}_{\mu_2\ldots\mu_D]} = \mathcal{F}\sqrt{-G}\varepsilon_{\mu_1\ldots\mu_D} \tag{21}$$

are the field-strengths of the electromagnetic $\mathcal{A}_\mu$ and Kalb-Ramond $\mathcal{A}_{\mu_1\ldots\mu_{D-1}}$ gauge potentials [31].

3 Lightlike Brane Dynamics in Various Types of Gravitational Backgrounds

World-volume reparametrization invariance allows us to introduce the standard synchronous gauge-fixing conditions:

$$\gamma^{0i} = 0 \;\; (i = 1,\ldots,p) \; , \; \gamma^{00} = -1 \; . \tag{22}$$

Also, we will use a natural ansatz for the "electric" part of the auxiliary world-volume gauge field-strength (6):

$$F^{*i} = 0 \;\; (i = 1,\ldots,p) \quad , \quad \text{i.e.} \;\; F_{0i_1\ldots i_{p-1}} = 0 \; , \tag{23}$$

meaning that we choose the lightlike direction in Eq.(10) to coincide with the brane proper-time direction on the world-volume ($F^{*\,a}\partial_a \sim \partial_\tau$). The Bianchi identity ($\nabla_a F^{*\,a} = 0$) together with (22)–(23) and the definition for the dual field-strength in (6) imply:

$$\partial_\tau \gamma^{(p)} = 0 \quad \text{where} \;\; \gamma^{(p)} \equiv \det\|\gamma_{ij}\| \; . \tag{24}$$

Taking into account (22)–(23), Eqs.(9) acquire the following gauge-fixed form (recall definition of the induced metric g_{ab} (5)):

$$g_{00} \equiv \dot{X}^\mu G_{\mu\nu}\dot{X}^\nu = 0 \quad , \quad g_{0i} = 0 \quad , \quad g_{ij} - 2a_0\,\gamma_{ij} = 0 \; , \tag{25}$$

where a_0 is the same constant as in (14).

3.1 Spherically Symmetric Backgrounds

Here we will be interested in static spherically symmetric solutions of Einstein-Maxwell equations (see Eqs.(35)–(36) below). We will consider the following generic form of static spherically symmetric metric:

$$ds^2 = -A(\eta)dt^2 + \frac{d\eta^2}{A(\eta)} + C(\eta)h_{ij}(\vec{\theta})d\theta^i d\theta^j \; , \quad (26)$$

or, in Eddington-Finkelstein coordinates [32] ($dt = dv - \frac{d\eta}{A(\eta)}$) :

$$ds^2 = -A(\eta)dv^2 + 2dv\, d\eta + C(\eta)h_{ij}(\vec{\theta})d\theta^i d\theta^j \; . \quad (27)$$

Here h_{ij} indicates the standard metric on the sphere S^p. The radial-like coordinate η will vary in general from $-\infty$ to $+\infty$.

We will consider the simplest ansatz for the *LL-brane* embedding coordinates:

$$X^0 \equiv v = \tau \quad , \quad X^1 \equiv \eta = \eta(\tau)$$
$$X^i \equiv \theta^i = \sigma^i \;\; (i = 1, \ldots, p) \; . \quad (28)$$

Now, the *LL-brane* equations (25) together with (24) yield:

$$-A(\eta) + 2\,\dot{\eta} = 0 \quad , \quad \partial_\tau C = \dot{\eta}\; \partial_\eta C \,\big|_{\eta=\eta(\tau)} = 0 \; . \quad (29)$$

First, we will consider the case of $C(\eta)$ as non-trivial function of η (*i.e.*, proper spherically symmetric space-time). In this case Eqs.(29) imply:

$$\dot{\eta} = 0 \;\to\; \eta(\tau) = \eta_0 = \text{const} \quad , \quad A(\eta_0) = 0 \; . \quad (30)$$

Eq.(30) tells us that consistency of *LL-brane* dynamics in a proper spherically symmetric gravitational background of codimension one requires the latter to possess a horizon (at some $\eta = \eta_0$), which is automatically occupied by the *LL-brane* ("horizon straddling" according to the terminology of Ref.[4]). Similar property – "horizon straddling", has been found also for *LL-branes* moving in rotating axially symmetric (Kerr or Kerr-Newman) and rotating cylindrically symmetric black hole backgrounds [12, 13].

With the embedding ansatz (28) and assuming the bulk Maxwell field to be purely electric static one ($\mathcal{F}_{0\eta} = \mathcal{F}_{v\eta} \neq 0$, the rest being zero; this is the relevant case to be discussed in what follows), Eq.(19) yields the simple relation: $\partial_i \chi = 0$, i.e. $\chi = \chi(\tau)$. Further, the only non-trivial contribution of the second order (w.r.t. world-volume proper time derivative) X^μ-equations of motion (20) arises for $\mu = v$, where the latter takes the form of an evolution equation for the dynamical tension $\chi(\tau)$. In the case of absence of couplings to bulk space-time gauge fields, the latter yields exponentional "inflation"/"deflation" at large times for the dynamical *LL-brane* tension:

$$\chi(\tau) = \chi_0 \exp\Big\{-\tau\Big(\frac{1}{2}\partial_\eta A + p a_0 \partial_\eta C\Big)_{\eta=\eta_0}\Big\} \; , \quad (31)$$

$\chi_0 = \text{const}$. Similarly to the "horizon straddling" property, exponential "inflation"/"deflation" for the *LL-brane* tension has also been found in the case of test *LL-brane* motion in rotating axially symmetric and rotating cylindrically symmetric black hole backgrounds (for details we refer to Refs.[11, 12, 13]). This phenomenon is an analog of the "mass inflation" effect around black hole horizons [15].

3.2 Product-Type Gravitational Backgrounds: Bertotti-Robinson Space-Time

Consider now the case $C(\eta) = \text{const}$ in (27), *i.e.*, the corresponding space-time manifold is of product type $\Sigma_2 \times S^p$. A physically relevant example is the Bertotti-Robinson [24, 25] space-time in $D = 4$ (*i.e.*, $p = 2$) with (non-extremal) metric (cf.[25]) :

$$ds^2 = r_0^2 \left[-\eta^2 dt^2 + \frac{d\eta^2}{\eta^2} + d\theta^2 + \sin^2\theta d\varphi^2\right] \; , \quad (32)$$

or in Eddington-Finkelstein (EF) form ($dt = \frac{1}{r_0^2}dv - \frac{d\eta}{\eta^2}$):

$$ds^2 = -\frac{\eta^2}{r_0^2}dv^2 + 2dv d\eta + r_0^2\left[d\theta^2 + \sin^2\theta d\varphi^2\right] \; . \quad (33)$$

At $\eta = 0$ the Bertotti-Robinson metric (32) (or (33)) possesses a horizon. Further, we will consider the case of Bertotti-Robinson universe with constant electric field $\mathcal{F}_{v\eta} = \pm\frac{1}{2r_0\sqrt{\pi}}$. In the present case the second Eq.(29) is trivially satisfied whereas the first one yields: $\eta(\tau) = \eta(0)\Big(1 - \tau\frac{\eta(0)}{2r_0^2}\Big)^{-1}$. In particular, if the *LL-brane* is initially (at $\tau = 0$) located on the Bertotti-Robinson horizon $\eta = 0$, it will stay there permanently.

4 Self-Consistent Wormhole Solutions Produced By Lightlike Branes

4.1 Lagrangian Formulation of Bulk Gravity-Matter System Coupled to Lightlike Brane

Let us now consider elf-consistent bulk Einstein-Maxwell-Kalb-Ramond system coupled to a charged

codimension-one *lightlike* p-brane (*i.e.*, $D = (p+1)+1$). It is described by the following action:

$$S = \int d^D x \sqrt{-G} \left[\frac{R(G)}{16\pi} - \frac{1}{4} \mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu} - \frac{1}{D!2} \mathcal{F}_{\mu_1 \ldots \mu_D} \mathcal{F}^{\mu_1 \ldots \mu_D} \right] + \widetilde{S}_{\rm LL} \; . \qquad (34)$$

Here $\mathcal{F}_{\mu\nu}$ and $\mathcal{F}_{\mu_1 \ldots \mu_D}$ are the Maxwell and Kalb-Ramond field-strengths (21). The last term on the r.h.s. of (34) indicates the reparametrization invariant world-volume action (18) of the *LL-brane* coupled to the bulk space-time gauge fields.

The pertinent Einstein-Maxwell-Kalb-Ramond equations of motion derived from the action (34) read:

$$R_{\mu\nu} - \frac{1}{2} G_{\mu\nu} R = 8\pi \left(T^{(EM)}_{\mu\nu} + T^{(KR)}_{\mu\nu} + T^{(brane)}_{\mu\nu} \right) \; , \qquad (35)$$

$$\partial_\nu \left(\sqrt{-G} \mathcal{F}^{\mu\nu} \right) + q \int d^{p+1}\sigma \, \delta^{(D)}\Bigl(x - X(\sigma) \Bigr) \times \varepsilon^{ab_1 \ldots b_p} F_{b_1 \ldots b_p} \partial_a X^\mu = 0 \; , \qquad (36)$$

$$\varepsilon^{\nu\mu_1 \ldots \mu_{p+1}} \partial_\nu \mathcal{F} - \beta \int d^{p+1}\sigma \, \delta^{(D)}(x - X(\sigma)) \times \varepsilon^{a_1 \ldots a_{p+1}} \partial_{a_1} X^{\mu_1} \ldots \partial_{a_{p+1}} X^{\mu_{p+1}} = 0 \; , \qquad (37)$$

where in the last equation we have used the last relation (21). The explicit form of the energy-momentum tensors read:

$$T^{(EM)}_{\mu\nu} = \mathcal{F}_{\mu\kappa}\mathcal{F}_{\nu\lambda} G^{\kappa\lambda} - G_{\mu\nu} \frac{1}{4} \mathcal{F}_{\rho\kappa}\mathcal{F}_{\sigma\lambda} G^{\rho\sigma} G^{\kappa\lambda} \; , \qquad (38)$$

$$T^{(KR)}_{\mu\nu} = \frac{1}{(D-1)!} \Bigl[\mathcal{F}_{\mu\lambda_1 \ldots \lambda_{D-1}} \mathcal{F}_\nu^{\;\lambda_1 \ldots \lambda_{D-1}} - \frac{1}{2D} G_{\mu\nu} \mathcal{F}_{\lambda_1 \ldots \lambda_D} \mathcal{F}^{\lambda_1 \ldots \lambda_D} \Bigr] = -\frac{1}{2} \mathcal{F}^2 G_{\mu\nu} \; , \qquad (39)$$

$$T^{(brane)}_{\mu\nu} = - G_{\mu\kappa} G_{\nu\lambda} \int d^{p+1}\sigma \, \frac{\delta^{(D)}\Bigl(x - X(\sigma) \Bigr)}{\sqrt{-G}} \times \chi \sqrt{-\gamma} \gamma^{ab} \partial_a X^\kappa \partial_b X^\lambda \; , \qquad (40)$$

where the brane stress-energy tensor is straightforwardly derived from the world-volume action (1) (or, equivalently, (7); recall $\chi \equiv \frac{\Phi}{\sqrt{-\gamma}}$ is the variable brane tension).

Using again the embedding ansatz (28) together with (30) as well as (22)–(25), the Kalb-Ramond equations of motion (37) reduce to:

$$\partial_\eta \mathcal{F} + \beta \delta(\eta - \eta_0) = 0 \qquad (41)$$

implying

$$\mathcal{F} = \mathcal{F}_{(+)} \theta(\eta - \eta_0) + \mathcal{F}_{(-)} \theta(\eta_0 - \eta) \qquad \mathcal{F}_{(\pm)} = \mathrm{const} \quad , \quad \mathcal{F}_{(-)} - \mathcal{F}_{(+)} = \beta \qquad (42)$$

Therefore, a space-time varying non-negative cosmological constant is dynamically generated in both exterior and interior regions w.r.t. the horizon at $\eta = \eta_0$ (cf. Eq.(39)): $\Lambda_{(\pm)} = 4\pi \mathcal{F}^2_{(\pm)}$. Hereafter we will discard the presence of the Kalb-Ramond gauge field and, correspondingly, there will be no dynamical generation of cosmological constant.

4.2 Asymmetric Wormholes

We will consider in what follows the case of $D = 4$-dimensional bulk space-time and, correspondingly, $p = 2$ for the *LL-brane*. For further simplification of the numerical constant factors we will choose the following specific ("wrong-sign" Maxwell) form for the Lagrangian of the auxiliary non-dynamical world-volume gauge field (6): $L(F^2) = \frac{1}{4} F^2 \;\; \rightarrow \;\; a_0 = M$, where again a_0 is the constant defined in (14) and M denotes the original integration constant in Eqs.(8).

We will seek a self-consistent solution of the equations of motion of the coupled Einstein-Maxwell-*LL-brane* system (Eqs.(35)–(36) and (8)–(9), (19)–(20)) describing an asymmetric wormhole space-time with spherically symmetric geometry. The general form of asymmetric wormhole metric (in Eddington-Finkelstein coordinates) reads:

$$ds^2 = -A(\eta) dv^2 + 2 dv d\eta + C(\eta) \left[d\theta^2 + \sin^2\theta d\varphi^2 \right] \; , \qquad (43)$$

$$A(0) = 0 \quad (\text{ "throat" at } \eta_0 = 0 \text{ }) \qquad A(\eta) > 0 \;\; \text{for all } \eta \neq 0 \; . \qquad (44)$$

The radial-like coordinate η varies from $-\infty$ to $+\infty$ and the metric coefficients $A(\eta)$ and $C(\eta)$ are continuous but *not necessarily differentiable* w.r.t. η at the wormhole "throat" $\eta = 0$. We will require:

$$\partial_\eta A \bigm|_{\eta \to +0} \equiv \partial_\eta A \bigm|_{+0} > 0 \; , \; \partial_\eta A \bigm|_{\eta \to -0} \equiv \partial_\eta A \bigm|_{-0} > 0 \; . \qquad (45)$$

Einstein equations (35) yield for the metric (43):

$$\partial_\eta A \bigm|_{+0} - \partial_\eta A \bigm|_{-0} = -16\pi\, \chi \qquad \partial_\eta \ln C \bigm|_{+0} - \partial_\eta \ln C \bigm|_{-0} = -\frac{4\pi\,\chi}{a_0} \; . \qquad (46)$$

For the *LL-brane* equations of motion we use again the embedding (28) resulting in the *LL-brane* "horizon straddling" (30). On the other hand, the second order Eqs.(20) contain "force" terms (the geodesic ones involving the Christoffel connection coefficients as well as those coming from the *LL-brane* coupling to the bulk Maxwell gauge field) which display discontinuities across the "throat" at $\eta = 0$ occupied by the *LL-brane* due to the delta-function

terms in the respective bulk space-time Einstein-Maxwell Eqs.(35)–(36) (now $\eta_0 \equiv 0$). The discontinuity problem is resolved following the approach in Ref.[3] (see also the regularization approach in Ref.[33], Appendix A) by taking mean values of the "force" terms across the discontinuity at $\eta = 0$. Furthermore, we will require $\chi = \text{const}$ (independent of the *LL-brane* proper time τ) for consistency with the matching relations (46). Thus, in the case of the *LL-brane* embedding (28) the X^μ-equation (20) for $\mu = v$ with $D = 4$, $p = 2$, no Kalb-Ramond coupling, *i.e.*, $\mathcal{F} = 0$, and using the gauge-fixing (22), becomes:

$$\chi\Big[\frac{1}{4}\Big(\partial_\eta A\,\big|_{+0} + \partial_\eta A\,\big|_{-0}\Big) + a_0\Big(\partial_\eta \ln C\,\big|_{+0} + \partial_\eta \ln C\,\big|_{-0}\Big)\Big] - q\sqrt{2a_0}\Big[\mathcal{F}_{v\eta}\,\big|_{+0} + \mathcal{F}_{v\eta}\,\big|_{-0}\Big] = 0 \tag{47}$$

In the present wormhole solution we will take "left" Bertotti-Robinson "universe" with:

$$A(\eta) = \frac{\eta^2}{r_0^2} \quad , \quad C(\eta) = r_0^2 \quad , \quad \mathcal{F}_{v\eta} = \pm\frac{1}{2\sqrt{\pi}\,r_0} \tag{48}$$

for $\eta < 0$, and "right" Reissner-Nordström "universe" with:

$$A(\eta) \equiv A_{\rm RN}(r_0+\eta) = 1 - \frac{2m}{r_0+\eta} + \frac{Q^2}{(r_0+\eta)^2} \;,$$

$$C(\eta) = (r_0+\eta)^2 \;,\; \mathcal{F}_{v\eta} \equiv \mathcal{F}_{vr}\big|_{RN} = \frac{Q}{\sqrt{4\pi}(r_0+\eta)^2} \;, \tag{49}$$

for $\eta > 0$, and

$$A(0) \equiv A_{\rm RN}(r_0) = 0 \;,\; \partial_\eta A\,\big|_{+0} \equiv \partial_r A_{\rm RN}\,\big|_{r=r_0} > 0 \tag{50}$$

where $\mathcal{F}_{v\eta}$'s are the respective Maxwell field-strengths and where $Q = r_0\Big[\sqrt{\frac{8\pi}{a_0}}\,q r_0 \pm 1\Big]$ is determined from the discontinuity of $\mathcal{F}_{v\eta}$ in Maxwell equations (36) across the charged *LL-brane*. Here we have used the standard coordinate notations for the Reissner-Nordström metric coefficients and Coulomb field strength:

$$A_{\rm RN}(r) = 1 - \frac{2m}{r} + \frac{Q^2}{r^2} \quad , \quad \mathcal{F}_{vr}\,\big|_{RN} = \frac{Q}{\sqrt{4\pi}r^2} \;. \tag{51}$$

Since obviously both Bertotti-Robinson (48) and Reissner-Nordström (49) metrics do satisfy the "vacuum" Einstein-Maxwell equations (Eqs.(35)–(36) *without* the *LL-brane* stress-energy tensor) it remains to check the matching of both metrics at the "throat" $\eta = 0$ (the location of the *LL-brane*) according to Eqs.(46)–(47). In this case the latter equations give:

$$\partial_r A_{\rm RN}\,\big|_{r=r_0} = -16\pi\,\chi \;\;,\;\; \partial_r \ln r^2\,\big|_{r=r_0} = -\frac{4\pi}{a_0}\chi \tag{52}$$

$$\chi\Big[\frac{1}{4}\partial_r A_{\rm RN}\,\big|_{r=r_0} + a_0 \partial_r \ln r^2\,\big|_{r=r_0}\Big] - 2q^2 \mp \frac{q}{r_0}\sqrt{\frac{2a_0}{\pi}} = 0 \;. \tag{53}$$

From (52)–(53) we get:

$$r_0 = \frac{a_0}{2\pi|\chi|} \quad , \quad m = \frac{a_0}{2\pi|\chi|}(1-4a_0) \;, \tag{54}$$

implying that the dynamical *LL-brane* tension χ must be negative, thus identifying the *LL-brane* as "exotic matter" [19, 21]. Further we obtain a quadratic equation for $|\chi|$:

$$\chi^2 + \frac{q^2}{4\pi} \pm \frac{q}{2\sqrt{2\pi\,a_0}}|\chi| = 0 \;, \tag{55}$$

which dictates that we have to choose the sign of q to be opposite to the sign in the expression for the Bertotti-Robinson constant electric field (last Eq.(48)). There are two positive solutions for $|\chi|$:

$$|\chi| = \frac{|q|}{4\sqrt{2\pi\,a_0}}\big(1 \pm \sqrt{1-8a_0}\big) \quad \text{for}\;\; a_0 < 1/8 \;. \tag{56}$$

Using (54) and (56) the expression for Q^2 reads:

$$Q^2 = \frac{a_0^2}{4\pi^2\chi^2}(1-8a_0) = \frac{8a_0^3}{\pi\,q^2}\,\frac{1-8a_0}{\big(1\pm\sqrt{1-8a_0}\big)^2} \tag{57}$$

Thus, we have constructed a solution to Einstein-Maxwell equations (35)–(36) in $D = 4$ describing a wormhole space-time manifold consisting of a "left" Bertotti-Robinson universe with two compactified space dimensions and a "right" Reissner-Nordström universe connected by a "throat" materialized by a *LL-brane*. The "throat" is a common horizon for both universes where for the "right" universe it is the external Reissner-Nordström horizon. All wormhole parameters, including the dynamical *LL-brane* tension, are determined in terms of the surface charge density q of the *LL-brane* (cf. Eq.(18)) and the integration constant a_0 (14) characterizing *LL-brane* dynamics in a bulk gravitational field.

5 Conclusions. Travel to Compactland Through a Wormhole

In this work we have explored the use of (codimension-one) *LL-branes* for construction of new

asymmetric wormhole solutions of Einstein-Maxwell equations. We have put strong emphasize on the crucial properties of the dynamics of *LL-branes* interacting with gravity and bulk space-time gauge fields:

(i) "Horizon straddling" – automatic position of the *LL-brane* on (one of) the horizon(s) of the bulk space-time geometry;

(ii) Intrinsic nature of the *LL-brane* tension as an additional *dynamical degree of freedom* unlike the case of standard Nambu-Goto p-branes;

(iii) The *LL-brane* stress-energy tensor is systematically derived from the underlying *LL-brane* Lagrangian action and provides the appropriate source term on the r.h.s. of Einstein equations to enable the existence of consistent non-trivial wormhole solutions;

(iv) Electrically charged *LL-branes* naturally produce *asymmetric* wormholes with the *LL-brane* itself materializing the wormhole "throat" and uniquely determining the pertinent wormhole parameters.

Finally, let us point out that the above asymmetric wormhole connecting Reissner-Nordström universe with a Bertotti-Robinson universe through a lightlike hypersurface occupied by a *LL-brane* is *traversable* w.r.t. the *proper time* of a traveling observer. The latter property is similar to the *proper time* traversability of other symmetric and asymmetric wormholes with *LL-brane* sitting on the "throat" [12, 13, 16, 17]. Indeed, let us consider test particle ("traveling observer") dynamics in the asymmetric wormhole background given by (48)–(49), which is described by the action:

$$S_{\rm particle} = \frac{1}{2}\int d\lambda \Big[\frac{1}{e}\,\dot{x}^\mu \dot{x}^\nu\, G_{\mu\nu} - e m_0^2\Big]\,. \qquad (58)$$

Using energy $\mathcal{E}$ and orbital momentum $\mathcal{J}$ conservation and introducing the *proper* world-line time s ($\frac{ds}{d\lambda} = e m_0$), the "mass-shell" equation (the equation w.r.t. the "einbein" e produced by the action (58)) yields:

$$\left(\frac{d\eta}{ds}\right)^2 + \mathcal{V}_{\rm eff}(\eta) = \frac{\mathcal{E}^2}{m_0^2}\,,\; \mathcal{V}_{\rm eff}(\eta) \equiv A(\eta)\Big(1+\frac{\mathcal{J}^2}{m_0^2 C(\eta)}\Big) \qquad (59)$$

with $\mathcal{A}(\eta)$, $C(\eta)$ – the same metric coefficients as in (48)–(50).

For generic values of the parameters the effective potential in the Bertotti-Robinson universe (48) (*i.e.*, for $\eta < 0$) has harmonic-oscillator-type form. Therefore, a traveling observer starting in the Reissner-Nordström universe (49) (*i.e.*, at some $\eta > 0$) and moving "radially" along the η-direction towards the horizon, will cross the wormhole "throat" ($\eta = 0$) within finite interval of his/her proper time, then will continue into the Bertotti-Robinson universe subject to harmonic-oscillator deceleration force, will reverse back at the turning point and finally will cross the "throat" back into the Reissner-Nordström universe.

Let us stress that, as in the case of the previously constructed symmetric and asymmetric wormholes via *LL-branes* sitting on their "throats" [12, 13, 16, 17], the present Reissner-Nordström-to-Bertotti-Robinson wormhole is *not* traversable w.r.t. the "laboratory" time of a static observer in either universe.

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